-: Numerical Problems on Simplex method :-

Problem ②: Solve the following problem by Simplex method.

Maximise $Z = 5x_1 + 4x_2$

Subjected to
$$6x_1 + 4x_2 \leq 24$$
$$x_1 + 2x_2 \leq 6$$
$$-x_1 + x_2 \leq 1$$
$$x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

Solution :-

Add the slack variable S_1, S_2, S_3, S_4 in the four constraint and remove in equalities.

$$6x_1 + 4x_2 + S_1 = 24 \quad \text{——} ①$$
$$x_1 + 2x_2 + S_2 = 6 \quad \text{——} ②$$
$$-x_1 + x_2 + S_3 = 1 \quad \text{——} ③$$
$$x_2 + S_4 = 2 \quad \text{——} ④$$
$$x_1, x_2, S_1, S_2, S_3, S_4 > 0$$

objective function $Z = 5x_1 + 4x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4$

	C_j	5	4	0	0	0	0		
e_i	BV	x_1	x_2	S_1	S_2	S_3	S_4	RHS	Ratio
0	S_1	[6] K.E.	4	1	0	0	0	24	4 ← key row
0	S_2	1	2	0	1	0	0	6	6
0	S_3	-1	1	0	0	1	0	1	-1
0	S_4	0	1	0	0	0	1	2	∞
	E_j	0	0	0	0	0	0		
	$C_j - E_j$	5	4	0	0	0	0		

↑ key column

	C_j	5	4	0	0	0	0			
C_i	BV	x_1	x_2	S_1	S_2	S_3	S_4	RHS		Ratio
5	x_1	1	$\frac{2}{3}$	$\frac{1}{6}$	0	0	0	4	$(R_1 \to \frac{R_1}{6})$	6
0	S_2	0	$\left[\frac{4}{3}\right]^{K.E}$	$-\frac{1}{6}$	1	0	0	2	$(R_2 \to R_2 - R_1)$	$\frac{3}{2} \leftarrow$ key row
0	S_3	0	5/3	$\frac{1}{6}$	0	0	0	5	$(R_3 \to R_3 + R_1)$	3
0	S_4	0	1	0	0	0	1	2		2
	E_j	5	$\frac{10}{3}$	$\frac{5}{6}$	0	0	0			
	$C_j - E_j$	0	$\frac{2}{3}$	$-\frac{5}{6}$	0	0	0			

↑ key column

	C_j	5	4	0	0	0	0		
C_i	BV	x_1	x_2	S_1	S_2	S_3	S_4	RHS	Ratio
5	x_1	1	0	$\frac{1}{4}$	$-\frac{1}{2}$	0	0	$3 \; (R_1 \to R_1 - \frac{2}{3} R_2)$	
4	x_2	0	1	$-\frac{1}{8}$	$\frac{3}{4}$	0	0	$\frac{3}{2} \; (R_2 \to \frac{3}{4} R_2)$	
0	S_3	0	0	$-\frac{1}{24}$	$-\frac{5}{4}$	0	0	$\frac{5}{2} \; (R_3 \to R_3 - \frac{5}{3} R_2)$	
0	S_4	0	0	$\frac{1}{8}$	$-\frac{3}{4}$	0	1	$\frac{1}{2} \; (R_4 \to R_4 - R_2)$	
	E_j	5	4	$\frac{3}{4}$	$\frac{1}{2}$	0	0		
	$C_j - E_j$	0	0	$-\frac{3}{4}$	$-\frac{1}{2}$	0	0		

We can see that all the values of $C_j - E_j$ row are either −ve or zero. Therefore optimal solution has been obtained.

Solution is $\boxed{x_1 = 3 \quad x_2 = \frac{3}{2}}$

$$Z_{max} = 5x_1 + 4x_2 = 5 \times 3 + 4 \times \frac{3}{2}$$

$$\boxed{Z_{max} = 21}$$

Solve the following problem by Simplex Method.

Problem ③:- Maximize $Z = 60x_1 + 70x_2$

subjected to

$$2x_1 + x_2 \leq 300$$
$$3x_1 + 4x_2 \leq 509$$
$$4x_1 + 7x_2 \leq 812$$
$$x_1, x_2 \geq 0$$

Ans

$x_1 = \frac{691}{5}$, $x_2 = \frac{118}{5}$ and

$Z_{max} = 9944$

Problem ④: Maximize $22x_1 + 6x_2 + 2x_3$

Subjected to
$$10x_1 + 2x_2 + x_3 \leq 100$$
$$7x_1 + 3x_2 + 2x_3 \leq 72$$
$$2x_1 + 4x_2 + x_3 \leq 80$$
$$x_1, x_2, x_3 \geq 0$$

Ans

$x_1 = \frac{73}{8}$, $x_2 = \frac{30}{8}$, $x_3 = \frac{177}{4}$

Problem ⑤ Maximize $Z = 40x_1 + 35x_2$

subjected to
$$2x_1 + 3x_2 \leq 60$$
$$4x_1 + 3x_2 \leq 96$$
$$x_1, x_2 \geq 0$$

Ans

$x_1 = 18$, $x_2 = 8$ and

$Z_{max} = 1000$

⇒ **Big M-Method** :- The basic simplex method is applied to LPP with less than or equal to type constraints.

The LPP can also be characterized by the presence of both "less than or equal to" type or "greater than or equal to" type constraints. In such case it is not always possible to obtain an initial basic feasible solution using slack variables.

* The greater than or equal to type of LPP can be solved by using Big M-method. In this method we need artificial variables for determining the initial basic feasible solution.

Let us take following problem to illustrate the Big M-method.

Problem ⓑ :- Minimize $z = 2x_1 + 3x_2$

subjected to $x_1 + x_2 \geq 5$

$x_1 + 2x_2 \geq 6$

$x_1, x_2 \geq 0$

Solution:- $z = 2x_1 + 3x_2$ (minimization)

Converting to standard form: Maximize $z = -2x_1 - 3x_2$

~~objecti~~ i.e. minimization problem is converted into the maximization problem by multiplying RHS of objective function by -1.

constraints $x_1 + x_2 - S_1 = 5$ ——(i)

$x_1 + 2x_2 - S_2 = 6$ ——(ii)

$x_1, x_2, S_1, S_2 \geq 0$

objective function $z = -2x_1 - 3x_2 + 0S_1 + 0S_2$

Here surplus variables S_1 & S_2 are sabstracted from constraints (i) & (ii).

Now x_1 and x_2 canbe taken as zero to get the basic variables S_1 & S_2. From this $S_1 = -5$ & $S_2 = -6$

This is infeasible solution as surplus variables S_1 & S_2 have -ve values. In order to over come this problem we add artificial variables A_1 & A_2 in equ. (i) & (ii)

$$x_1 + x_2 - S_1 + A_1 = 5 \quad \text{—(iii)}$$

$$x_1 + 2x_2 - S_2 + A_2 = 6 \quad \text{—(iv)}$$

$$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$$

and objective function

Maximize $Z' = -2x_1 - 3x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$

where M is a very large +ve number. Purpose for this is that the artificial variables initially appear in the starting basic solution.

	Cj	-2	-3	0	0	-M	-M		
ei	BV	x_1	x_2	S_1	S_2	A_1	A_2	RHS	Ratio
-M	A_1	1	1	-1	0	1	0	5	5
-M	A_2	1	[2] K.E.	0	-1	0	1	6	3 ← Key row
	Ej	-2M	-3M	M	M	-M	-M		
	Cj-Ej	-2+2M	-3+3M	-M	-M	0	0		
			↑ Keycolumn						

	C_j	-2	-3	0	0	$-M$	$\cancel{-M}$		
C_i	B.V	x_1	x_2	S_1	S_2	A_1	$\cancel{A_2}$ — leaving element	RHS	Ratio
$-M$	A_1	$\left[\frac{1}{2}\right]^{K.E.}$	0	-1	$\frac{1}{2}$	1	$\cancel{\frac{1}{2}}$	2 $(R_1 \to R_1 - R_2)$	$4 \leftarrow$
-3	x_2	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	$\cancel{\frac{1}{2}}$	3 $(R_2 \to \frac{R_2}{2})$	6
	E_j	$-\frac{M}{2}-\frac{3}{2}$	-3	M	$-\frac{M}{2}+\frac{3}{2}$	$-M$	$\cancel{-\frac{M}{2}-\frac{3}{2}}$		
	$C_j - E_j$	$+\frac{M}{2}-\frac{1}{2}$	0	$-M$	$\frac{M}{2}-\frac{3}{2}$	0			

↑ Key column

	C_j	-2	-3	0	0	
C_i	BV	x_1	x_2	S_1	S_2	RHS
-2	x_1	1	0	-2	1	4 $(R_1 \to R_1 \times 2)$
-3	x_2	0	1	1	-1	1 $(R_2 \to R_2 - \frac{R_1}{2})$
	E_j	-2	-3	1	1	
	$C_j - E_j$	0	0	-1	-1	

All the terms in $C_j - E_j$ are either zero or -ve

hence optimal solution has reached.

Final solution $\boxed{\begin{matrix} x_1 = 4 \\ x_2 = 1 \end{matrix}}$

$$Z_{min} = 2x_1 + 3x_2 = 2 \times 4 + 3 \times 1 = 11$$

⇒ Unbounded Solution :- A LPP is said to have unbounded solution when the ratio column, we get all entries either zero or -ve and there is no +ve entry. This indicates that the value of entering variable selected from the key column can be as large as we like without violating the feasible conditions and the problem is said to have unbounded solution.

⇒ Infinite number of solution :- A LPP is said to have infinite number of solutions if during any iteration in $C_j - E_j$ row, we have all the terms either zero or -ve. It shows that optimum solution has reached. But since one of the regular variables has zero value in $C_j - E_j$ row, it can be concluded that there exists an alternative optimal solution.

⇒ No feasible solution :- In some LPP it can be seen that while solving problem with artificial variables, $C_j - E_j$ row shows that optimal solution is reached whereas we still have artificial variables in the current solution having some +ve value. In such situations, it can be concluded that problem does not have any feasible solution at all.